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PRICE AND YIELD CALCULATIONS ON GOVERNMENT OF CANADA DOMESTIC SECURITIES

Introduction

This note describes the methods used at the Bank of Canada to calculate prices and yields on domestic Government of Canada marketable securities.[1] There are two broad classes of such instruments; treasury bills and Canada bonds.

Treasury bills are money market instruments that are auctioned weekly (usually on Tuesday for Thursday settlement) by the Bank of Canada[2] in maturities of 3, 6 and 12 months[3], although shorter bills are auctioned occasionally for government cash management purposes. Canadian treasury bills are very similar to US Treasury bills. They do not carry a specified contractual rate of interest but are issued and trade at a discount to par value, with the return to the purchaser being based on the difference between the purchase price and the sale price (or par value if held to maturity). Most trading is done before noon for same-day settlement, although some next-day settlement trading occurs in the afternoon.[4]

Canada bonds[5] are issued by the Government of Canada in maturities ranging from two to thirty years. Canada bonds are very similar to US Treasury bonds; they have a fixed maturity date, pay a fixed semi-annual coupon and are quoted as a percent of par value net of accrued interest. In the past, the Government has issued bonds with various option features, including extendible bonds[6] as recently as 1982, and callable bonds[7] in 1956. Also, "purchase fund" bonds were issued in the mid- to late-seventies, that require the government to make periodic partial redemptions before maturity, where such repayments are made through open market purchases over the lives of the bonds. [8] In secondary market trading, normal settlement is two business days for issues that mature up to three years from the trade date, and three days for issues that are longer than three years.

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1 "Domestic" securities are denominated in Canadian dollars and trade within Canada. The methodologies outlined in this note are not purported to be "official" in the same way as those given for U.S. securities in the Securities Industry Association's Standard Securities Calculation Methods. Although believed to be consistent with general Canadian market practice, some market participants may use different methodologies from those presented here. Neither the Bank of Canada nor the author of this note shall bear any liability whatso ever for any errors which may be in or be the result of the methodologies described in this note.

2 The Bank of Canada acts as fiscal agent for the Government of Canada. In this capacity it advises the Government on its debt management strategy, and it manages the issuance and maintenance of all of the Government's domestic debt issues.

3 If the normal Thursday maturity date falls on a holiday, the maturity date is usually moved to the prior business day.

4 For a more detailed discussion of the Canadian treasury bill market, see K.Fettig,"The Government of Canada treasury bill market and its role in monetary policy", Bank of Canada Review, Spring 1994.

5 Although not discussed in this note, the government also issues inflation-index-linked bonds, called "real return bonds". For a more complete discussion of the Canada bond market, see A.Branion,"The Government of Canada bond market since 1980", Bank of Canada Review, Fall 1995. For more information on real return bonds, see A.Cote, J.Jacob, J.Nelmes and M.Whittingham,"Inflation expectations and real return bonds", Bank of Canada Review, forthcoming.

6 Extendible bonds are short maturity bonds that are exchangeable into long maturity bonds at the holder's option. For example, the 7.50% Canadas due October 1, 1979 were exchangeable into an equal par value of 8.75% bonds due October 1, 1984.

7 Callable bonds may be called in for redemption by the issuer before maturity. For example, the 3.75% Canadas issued September 15, 1956 and due March 15, 1998 are callable on 60 days notice at any time after September 15, 1996.

8 With the added stipulation that all purchase fund purchases be below the bond's original issue price. Since all of the purchase fund bonds have been recently trading well above their issue prices, the purchase fund has recently been inactive.

Treasury Bill Price and Yield Calculations

Government of Canada treasury bill yields and prices are calculated on a simple interest basis using an "actual/365" day calendar[9] (instead of the US "actual/360"). Hence, the price (per $100 par value) of a *T*-day Canadian treasury bill yielding *Y* would be calculated as follows[10]:

$$P = 100/(1 + Y * T/365) \quad (1)$$

The yield formula is simply formula (1) solved for yield:

$$Y = (365/T)((100 - P)/P) \quad (2)$$

Short Canada Bond Price and Yield Calculations

The invoice price of a Canada bond is the sum of the quoted price plus accrued interest. Conceptually, accrued interest represents the portion of the next coupon payment which will be received by the purchaser, but has actually been earned by the seller of the bond. Accrued interest on Canada bonds is paid on an "actual/365" basis, so that interest accrues at the rate of 1/365th of the annual coupon rate per day, even in a leap year. The exception to the 1/365th accrual rule occurs when calculating more than 182 days of accrued interest, in which case accrued interest is calculated using 181.5 days. This exception accomodates the situation in which the accrued interest could exceed the amount of the forthcoming coupon payment.

Hence, the accrued interest for a bond that pays interest semi-annually at an annual rate of *C* (as a percent of par) is calculated as follows:

$$A = C(D/365) \quad (3)$$

where *D* is the number of accrued days to the settlement date (from but excluding the last coupon payment date).

When a bond has only one coupon remaining to be paid, it is market convention to calculate the yield on a simple interest basis over the fractional semi-annual period remaining to maturity on an "actual/365" basis. Hence, the price of a bond with *T* days remaining to maturity[11] to yield *Y* is:

$$P = (100 + C/2)/(1 + Y * T/365) - A \quad (4)$$

The yield formula is simply formula (4) solved for yield:

$$Y = (365/T)((100 + C/2)/(P + A) - 1) \quad (5)$$

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9 An "actual/365" basis uses the actual number of days to count the number of days between the settlement date and the maturity date, but the year is always assumed to be 365 days, irregardless of whether or not it is a leap year.

10 US Treasury bills trade on a "discount" basis, for which the price of an *T*-day bill trading at a discount yield of *D* is: *P = 100*(1 - T*D/360)*.

11 For short Canadas (with only one coupon payment remaining to maturity) the day count is done to the date on which the redemption proceeds are actually receivable. For example, if the bond matures on a weekend or holiday, the day count is done to the next business day. This may sound rather obvious, but for longer Canadas all date calculations ignore this nuance - if a payment falls on a Saturday, the date calculations are done to the Saturday.

Long Canada Bond Price and Yield Calculations

When more than one coupon payment remains to maturity, the standard US Treasury coupon security "street" method is used to calculate prices and yields.[12] This means, for example, that accrued interest is calculated using an "actual/actual" day count basis, even though the "actual/365" day count basis is still used for money settlement. The "actual/actual" basis means that the amount of daily accrual will depend upon the number of days in the relevant coupon period. For example, for the purchase of a 6 per cent bond maturing on July 15 to settle on May 15, the coupon interest will accrue at the rate of 1/181th of the 3 per cent semi-annual coupon rate per day (or 1/182th in a leap year), the inverse of the number of days from the last coupon payment (January 15) to the next (July 15). In this case, the amount of the accrual will be based on the number of days from that last coupon payment to the May 15 settlement date (120 or 121 days depending upon whether it is a leap year). However, the calculations are slightly different for "short" coupon Canadas.[13] When the next coupon payment is "short" the daily accrual is based upon the number of days from when the coupon would have been paid if it had been outstanding for some time, and the accrued interest is calculated from the issue date.[14] Continuing with the example, if the bonds had been issued on February 15, the amount of the daily accrual would still be the 1/181th or 182th of the 3 per cent annual rate per day, but the accrual calculation would start at February 15. Hence, the number of days of accrual to the May 15 settlement date would be 89 or 90 days.

Hence, the "actual/actual" basis accrued interest that is used in the price and yield calculations for a "long" Canada bond is calculated as follows:

$$A^* = (C/2)(D/E) \tag{6}$$

where D is the number of accrued days to the settlement date (from but excluding the last coupon payment date for a regular coupon or from but excluding the issue date for a short coupon) and E is the number of days in the coupon period (to the next coupon payment date from but excluding the date on which the last coupon payment would normally have been made).

Also, when a bond has more than one coupon payment remaining to maturity, yield is calculated on a semi-annually compounded "actual/actual" basis. Hence, the price of a Canada bond with N semi-annual coupon payments remaining to maturity to yield Y percent is:

$$P = C_1(1 + Y/2)^{-T_1} + (C/2)\sum_{k=2}^{N}(1 + Y/2)^{-T_k} + 100(1 + Y/2)^{-T_N} - A^* \tag{7}$$

where T_k is the fraction of a payment period to the kth payment date[15], and C_1 is the next coupon payment amount (as a per cent of par).

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12 The standard reference for US "street" calculations is J.Lynch and J.Mayle, Standard Securities Calculation Methods, Securities Industry Association. The US Treasury security calculation uses an "actual/actual" day count.

13 A new issue bond's first coupon payment is said to be "short" when the issue date is less than six full months from the first coupon payment date.

14 Another nuance sometimes occurs when an older bond's settlement date falls within its "closed-book" period, an approximately two-week period preceding each coupon payment date, in which case the seller of a "physical" bond remains the holder of record and receives the coupon payment. In this event, accrued interest is calculated and paid conventionally, but the seller gives the buyer a post-dated check for the amount of the coupon payment. However, few bonds now trade in physical form, so this closed-book nuance is seldom relevant. Since December 1993, all new issues of Canada bonds have been issued in fully-registered form through the Canadian Depository for Securities (CDS) book-based Debt Clearing Service, and all new issues since October 1, 1995 must be transferred through CDS, thereby eliminating the closed-book period for those issues.

15 $T_k = T_1 + k - 1$, where $T_1 = DSC/E$, where DSC is the number of days from but excluding the settlement date to the next coupon payment date.

The first coupon payment is usually just half the annual payment rate ($C_1 = C/2$) but when the next coupon payment is "short" $C_1 = (C/2)(DIC/E)$ where DIC is the number of days from but excluding the issue date to the first coupon payment date.

For purposes of calculation efficiency, the following formula can be used instead of formula (6) to calculate the price of Canada bonds with more than one coupon payment remaining to maturity. Basically, formula (8) compresses the sum of the geometric series (the Σ term) into an expression that requires no iteration[16], values the bond at the next coupon payment date (including the coupon then payable), discounts it back to the settlement date and subtracts the accrued interest:

$$P = (P' + C_1)(1 + Y/2)^{-T_1} - A \quad (8)$$

where P' is the net-of-accrued-interest value of the bond at the next coupon payment date:

$$P' = (C/Y) + (100 - C/Y)(1 + Y/2)^{1-N} \quad (9)$$

There is no closed-form solution for solving formulas (7) or (8) for yield, but they can be solved iteratively[17].

It is not uncommon for market participants to factor into the yield calculation the effect of "bad days" (when coupon or principal payments fall on non-business days). The Bloomberg yield calculator, for example, calculates a "true yield" which calculates the yield with all payment dates moved from weekends and holidays to the next business day. The impact of this calculation can be quite substantial, particularly at the short end of the maturity spectrum. However, the Bank does not currently accomodate bad days in its bond yield/price calculations.

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$$\sum_{k=2}^{N} (1 + Y/2)^{1-k} = (1 - (1 + Y/2)^{1-N})/(Y/2)$$

17 See page 109 of the Lynch and Mayle bokk, for example.

APPENDIX

CALCULATION EXAMPLES

Treasury bills

The following example illustrates the calculation of the price of a Treasury bill on a given yield using equation (1):

Settlement date	**August 8, 1996**
Maturity date	**January 30, 1997**
Term to maturity (*T*)	**175 days**
Yield to maturity (*Y*)	**4.000%**
Price as a per cent of par (*P*)	**100/(1+.04000(175/365)) = 98.11828**

Short Canada bonds

The following example illustrates the calculation of the price of a Canada bond on a given yield using equation (4) when only one coupon payment reamains until maturity:

Settlement date	**August 14, 1996**
Coupon rate as a per cent of par	**3.000**
Maturity date	**September 15, 1996**
Accrual days (*D*)	**Mar. 15, 1996 to Aug. 14, 1996 = 152 days**
Accrued interest (*A*)	**3.000(152/365) = 1.24931507**
Days to actual receipt of maturity proceeds (*T*)	**Aug. 14, 1996 to Sep. 16, 1996 = 33 days**
Yield to maturity (Y)	**15.000%**
Price (P) (Equation (4))	**(100+3.000/2)/(1+0.15000(33/365)) - 1.24931507 = 98.89259600**

Long Canada bonds

The following example illustrates the calculation of the price of a short coupon Canada bond on a given yield using equation (8) when more than one coupon payment remains until maturity:

Settlement date	**May 15, 1996**
Coupon rate as a per cent of par	**7.000**
Maturity date	**December 1, 2006**
Issue date	**February 15, 1996**
Last coupon payment date	**December 1, 1995**
Next coupon payment date	**June 1, 1996**
Accrual days (*D*)	**Feb. 15, 1996 to May 15, 1996 = 90 days**
Number of days in coupon period (*E*)	**Dec. 01, 1995 to June 1, 1996 = 183 days**
Accrued interest (A^*)	**(7.000/2)(90/183) = 1.72131148**
Days from issue to first coupon (*DIC*)	**Feb. 15, 1996 to June 1, 1996 = 107 days**
First coupon payment (C_1)	**(7.000/2)(107/183) = 2.04644809**
Number of coupon payments	**22**
Days from settlement to next coupon (DSC)	**17**
Fractional coupon period (T_1)	**0.092896**
Yield to maturity (*Y*)	**15.000%**
Value at next coupon payment date (*P'*)	**$(7.00/0.15)+(100-7.00/0.15)(1+0.15/2)^{1-N}$ = 58.34607866**
Add next coupon payment (C_1) and discount back to settlement	**$(58.34607866+2.04644809)(1+0.15/2)^{-0.092896}$ = 59.98815075**
Subtract accrued interest to arrive at price (*P*)	**59.98815075 - 1.72131148 = 58.26683927**